On Chemical Medicine, Thermodynamics, and Homeopathy

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ABSTRACT

The author indicates why homeopathic medicine is an example of future information medicine, a member of the more general psychophysiologic medicine group. Using standard chemical thermodynamics, it is readily shown that the driving force for all chemical reactions involves the logarithm of chemical activities for the different species involved. Because chemical activity is given by the product of concentration and thermodynamic activity coefficient, such reaction driving forces involve the sum of \( \ln \gamma_j \) and \( \ln c_j \) for the \( j \)-species. Homeopathy involves the dilution of \( c_j \) and succussion, which can increase \( \gamma_j \); thus, when \( c_j \) goes to the ultradilution state, the thermodynamic driving force for change does not disappear as is assumed by many and, in fact, can even increase through the \( \ln \gamma_j \) terms. Going to a more complex reference frame for viewing nature, one can, at least, qualitatively show how oscillating and decaying properties in time can occur for homeopathic remedies.

INTRODUCTION

Begin by considering what this author calls his “silver colloid” metaphor because it delineates three different kinds of medicine.

If one takes a beaker of water with some bacteria in it and then shakes some silver (Ag) colloid particles into the water, the bacteria will probably be killed via the bactericidal action of the Ag particles. The general conclusion drawn from this observation is that the physical contact between Ag and the bacterium is a necessary condition for killing of bacteria. This, in turn, has led to the assumption that pharmaceuticals do their work in the human body via contact-types of chemical reactions, and this has led to what is labeled as today’s chemical medicine.

What most people do not know is that, if one takes a fluorescent tube held horizontal and places silver colloid particles in it and then focuses the output light from the ignited tube onto the beaker of water containing bacteria, one also kills the bacteria.\(^1\) Such an experiment shows that it is not the physical contact between Ag and the bacterium that is necessary for the killing process to occur. Rather, it is one or more different types of photons from the electromagnetic (EM) emission spectrum of Ag, that entangle with the EM carrier wave from the fluorescent tube and are transported to the beaker of water that are the actual killing mechanism involved in the demise of the bacteria. Pursuing this line of research will inevitably lead to tomorrow’s EM medicine.

Over the past few years, this author and his colleagues have shown that one can imbibe a specific intention, from a deep meditative state, into a simple electronic device and have that device, in turn, “condition” a laboratory space wherein the proper experiment is running to test the efficacy of this intention procedure. This procedure has been successful with four uniquely different target experiments: (1) increase the pH of highly purified water by one pH unit, (2) decrease the pH of the same type of water by one pH unit, (3) increase the \textit{in vitro} thermodynamic activity of the liver enzyme alkaline phosphatase (ALP) by \( \sim 25\% \) at \( p < 0.001 \), and (4) increase the \textit{in vivo} \([\text{ATP}]/[\text{ADP}]\) ratio in the cells of fruit fly larvae by \( \sim 15\% \) at \( p < 0.001 \) to make them more physically fit and significantly reduce the larval development time to the adult fly stage.\(^2\) Replication of the first of these target experiments at 10 other laboratories in the United States and Europe\(^3\)–\(^6\) shows that this is a viable procedure that will ultimately lead to the day-after-tomorrow’s information medicine. By this labeling, it will come into common practice and usage.

\(^1\) The William A. Tiller Foundation for New Science, Payson, AZ.

\(^2\) The William A. Tiller Foundation for New Science, Payson, AZ.
In all three categories of medicine mentioned, one must not forget that human consciousness is involved both from the medical practitioner’s end and the patient’s end. Thus, in practice, they all fall in the general category of psychophysiologic medicine, in which one can expect that both unconscious and conscious expectations of either or both practitioner and/or patient can significantly influence the outcome of the treatment modality.

Today’s cold-laser therapy is an example of a combined EM / information medicine, in which one, in principle, delivers periodic bursts of EM or higher-dimensional information on an EM carrier wave. The lower is the frequency of the carrier wave, the deeper it penetrates into the body. The length of a burst, the spacing between bursts, and the frequency of the EM waves within the bursts all contribute to the time rate of flow of information into the body. Considering today’s homeopathic medicine in this list of medicine types, it probably belongs in the information medicine category even though it has been practiced for ~200 years.

Unfortunately, the viability of homeopathic medicine as a medical treatment modality produces a serious “boggle effect” in the minds of both chemists and allopathic medicine practitioners. Their mind-boggling question is, “How can there possibly be a chemical reactivity effect when the aqueous solution concentration of the treatment molecule is less than (Avogadro’s number)$^{-1}$?” However, based on standard thermodynamics, it should not be mind-boggling. The whys and wherefores of this are explored in the next section.

A brief reminder about some aspects of chemistry

For the detailed mathematical perspective of this section, please see Appendix I. Here, the author attempts to express the essence of Appendix I in largely nonmathematical language.

All processes in nature appear to be driven by differences in thermodynamic free energy functions that involve energy (enthalpy), entropy, and temperature. For chemical reactions between multiple species, the free energy change, $\Delta G_0$, defines the reaction at thermodynamic equilibrium. As such, it is always given by a relationship between the natural logarithms of the various equilibrium chemical activities, $a_j^e$, for the j’th species (see Equation I-2a of Appendix I). The actual thermodynamic driving force for change, $\Delta G$, is given in terms of $\Delta G_0$ and the same logarithm relationship between the actual chemical activities. However, it is the definition of $a_j$ that is important here; i.e.,

$$a_j = \gamma_j c_j \quad (\text{Equation 1a})$$

and

$$\ln a_j = \ln \gamma_j + \ln c_j \quad (\text{Equation 1b})$$

Here, $c_j$ = the concentration of j-species in the solvent, $\gamma_j$ = the thermodynamic activity coefficient of the j-species and $\ln$ is the natural logarithm. As such, $\gamma_j$ relates to the sum of all the environmental effects stored in the solvent that act on this j-type of molecule. Such environmental effects could be electric field, $E$, and magnetic field, $H$, effects or a wide variety of anomalous chemical potential effects. In most chemical texts, it is assumed that, as the chemical, concentration of j goes to very small values, the solution becomes an ideal solution, so $\gamma_j \rightarrow 1$. However, this need not be so when special environmental thermodynamic effects have been mathematically convoluted into a modified activity coefficient, $\hat{\gamma}_j$ (see Appendix I, Equations 1 to 4).

Homeopathic remedy preparation

In homeopathic remedy preparation, one does two things: (1) one sequentially dilutes the solution of j-species; that is, reduces $c_j$ and (2) one, simultaneously, sequentially successess the solution; that is, one alters $\gamma_j$ via the succussion process and, because a specific intention underlies this process, the infrastructure stored in the solvent can increase significantly. Thus, instead of Equation I-1 in Appendix I, one has

$$\text{water} + c_j \rightarrow \text{water}^* + c_j' \quad (\text{Equation 2a})$$

and

$$a_j \rightarrow a_j' \quad (\text{Equation 2b})$$

where $c_j$ is reduced to $c_j'$ by dilution, whereas water goes to water* by succussion and dilution. The most compelling message to note from Equations 2 is that, even when $c_j$ drops below one j-molecule per cc via dilution, $\gamma_j$ can increase significantly via intention-directed succussion. Thus, from Equation 1b, ln $a_j$ can increase significantly even when ln $c_j$ is negative. Both chemists and allopathic medical practitioners tend to focus their attention on the ln $c_j$ term and generally neglect the environmental information storage latent in the ln $\gamma_j$ term, and yet a standard thermodynamic treatment says it can become the dominant term as dilution continues.

In the Roy et al. study they macroscopic infrastructure of water is found to be very heterogeneous on the size scale of colloids and various processing treatments can lead to significant changes in such microstructural elements, that are still much larger than molecular sizes. Thus, one can surmise that $\gamma_j$-effects would change appreciably with water processing treatment. Elia and Niccoli, on the other hand, have made physicochemical measurements on diluted and succussed solutions and have observed strong excesses of exothermic heat release associated with the succussion process in comparison with just diluted control solutions. In certain cases studied, the magnitude of excess exothermic heat release increased with delay-time before testing after the actual treatment ended. This is very anomalous and indicates that something special is going on with $\gamma_j$. Thermoluminescence studies of ultrahigh dilutions of both sodium chloride and lithium chloride in solid-
ified and irradiated D₂O by Rey¹⁰ showed that, despite their dilution beyond the Avogadro number limit for these species, the emitted light was specific to these initially dissolved species. Thus, their specific initial presence and processing left a correlative memory element in the host material that was detectable even after the ultradilution process. Once again, this should be interpreted as a γ-effect.

Turning to the author’s and his colleagues’ replicated experimental research on intention to increase the pH of highly purified water in equilibrium with air within a device-conditioned laboratory by one pH unit using pH-electrodes with a measurement accuracy of ±0.01 pH units, the measuring equipment began to access data, not only from the normal, electrical atom/molecule level of physical reality, but also from a second, unique level of physical reality. It is this second level, rather than the first, that is influenced by human intention. Thus, the pH-measurement data³–⁶ were experimentally shown to yield:

\[
pH(t) = pH₁ + \gamma(t) pH₂(t) \quad (Equation \ 3)
\]

where \( pH₁ \) is the readily calculated value for level one, \( pH₂ \) is the experimentally registerable value for level two, \( \gamma \) is the effective value of the coupling coefficient, 0 < \( \gamma \) < 1, between these two levels of physical reality and \( t \) = time. In Equation 3, by definition, there is:

\[
pH₁ = \log_{10} a_{H^+} = \log_{10} \gamma_{H^+} C_{H^+} \quad (Equation \ 4a)
\]

so, from Equation 3:

\[
pH(t) = \log_{10} \gamma_{H^+} C_{H^+} \quad (Equation \ 4b)
\]

via a simple mathematical manipulation illustrated at the end of Appendix I. \( \gamma_{H^+} \) is the modified thermodynamic activity coefficient for the aqueous \( H^+ \) ion in a partially conditioned space. If \( \gamma \) ~ 0, then \( \gamma_{H^+} \rightarrow \gamma_{H^+} \), as expected. Thus, looked at from a simple spacetime perspective, the effect of this second level of physical reality on pH would be seen as a modified thermodynamic activity coefficient for \( H^+ \), as a modified environmental effect on \( \gamma_{H^+} \). Once again, one can see how truly remarkable new physics can be interpreted as a modified thermodynamic chemical activity coefficient from a conventional perspective.

For the interests here, experimentalists would benefit by measuring pH as the solution is diluted and succussed and plotting the pH versus \( \log_{10} C_{H^+} \) to extract the \( \log_{10} (\gamma_{H^+}(t)) \) contribution associated with succussion over a very long range of dilutions. This should be quite revealing because the high purity water used as the diluent is expected to have a pH given by pH₁ from Equation 3.

**Digging a little deeper**

There are two important aspects of thermodynamics that future homeopathic science must fully take into account. The first is that uniquely identifiable positive energy states in condensed matter contribute configurational entropy to the system and thus lower the total thermodynamic free energy, \( G \), of the system. Figure 1A illustrates how \( G \) is reduced by increase in the concentration, \( c_j \), of the j-th distinguishably different defect up to its equilibrium concentration \( c_j^* \). A similar lowering of \( G \) occurs for each different, uniquely distinguishable j-specified.

The chemical potential of a j-species drives its reactions and is given by the slope, with respect to \( j \), of \( G \) in Figure 1A and this is given by Figure 1B. This shows that: (1) \( \mu_j \) goes to \( -\infty \) as \( c_j \) approaches zero, (2) \( \mu_j \) has a fairly substantial region of negative values, \( 0 < c_j < c_j^* \), (3) \( \mu_j \) has a very large region of positive values, \( c_j^* < c_j < 1 \), and (4) \( \mu_j \) has an equilibrium value, \( c_j^* \), where \( \mu_j = 0 \); that is, the configurational entropy of \( j \) (multiplied by the absolute temperature) exactly balances the heat of solution resulting from adding \( j \) to the solution.

Nature really abhors large negative values of \( \mu_j \) and will do anything kinetically possible (like dissolving silica from the walls of the succussion vessel) to change \( \mu_j \) to a zero value. At any negative thermodynamic driving force, \( c_j < c_j^* \), \( \Delta \mu_j \) is given by:

\[
\Delta \mu_j = kT \ln(c_j/c_j^*) ; \ c_j < c_j^* \quad (Equation \ 5)
\]

At room temperature, \( kT \sim 2.6 \times 10^{-2} \) electron volts (eV) so that, when \( c_j^*/c_j \sim 10^{+16.7} \), \( \Delta \mu_j = -1 \) eV. In contrast, a single hydrogen bond in water has an average strength of \( \sim 0.22 \) eV (\( \sim 5 \) cal/mole). This can be readily provided when \( c_j^*/c_j \) is larger than \( 10^{+3.7} \).

The second important aspect to recognize is that, because of Equation 3, one must recognize that there are two levels of physical reality that one must deal with when the phenomenon under study involves the introduction of significant human consciousness. The reference frame (RF) that was developed to enhance understanding and the correlation between the physics of these two levels of physical reality is a duplex RF consisting of two, four-dimen-

![FIG. 1](image-url)
sional, reciprocal subspaces, one of which is spacetime (see Fig. 2).

This duplex RF is a special member of a general eight-dimensional space. Because these two subspaces are reciprocals of one another, the magnitude of any quality at one level and the magnitude of its equilibrium conjugate quality at the reciprocal subspace level are given by Fourier transform pair relationships\textsuperscript{13} that are somewhat modified by the coupling substance that gives rise to $z_{\text{eff}}$ in Equation 3. These two subspaces have been labeled direct space or D-space and reciprocal space or R-space. The coupler substance, needed because indications suggest that R-space substance is magnetic in nature, exists in the physical vacuum and travels faster than EM light, has been labeled “deltrons” from a higher dimension (beyond relativistic constraints). Figure 3 illustrates how such postulated deltrons could bring about meaningful interaction between slower than EM light, electric particles of normal physical reality with faster than EM light, magnetic information wave substance from the vacuum level of physical reality.

From an overly simplistic viewpoint, one could say that, for the past $\sim$400 years, establishment science has dealt with multiple aspects of the reaction equation:

\[
\text{MASS} \leftrightarrow \text{ENERGY} \quad (\text{Equation 6a})
\]

with one quality convertible to the other via Einstein’s $E = mc^2$ equation. This research shows that psychoenergetic science, which includes homeopathy, must deal with an expanded reaction equation:

\[
\text{MASS} \leftrightarrow \text{ENERGY} \leftrightarrow \text{CONSCIOUSNESS} \quad (\text{Equation 6b})
\]

although the word “consciousness” here is used in a way quite different than the normal dictionary usage of awareness, awakeness, and so on. Here, every term is considered to be convertible to another even though the quantitative connection to energy is not yet known. Perhaps, instead of asking what consciousness means, one should ask what consciousness does. When this is done, one realizes that consciousness manipulates information in the form of at least numbers, alphabet letters and, most generally, symbols. Thus, Equation 6b becomes more acceptable when it is phrased in the format

\[
\text{MASS} \leftrightarrow \text{ENERGY} \leftrightarrow \text{INFORMATION} \leftrightarrow \text{CONSCIOUSNESS} \quad (\text{Equation 6c})
\]

For the past 50 years or so, establishment science has recognized a quantitative relationship between information in units of “bits” and negative entropy.\textsuperscript{14} Thus, one sees that information has a respectable place in the thermodynamic framework as a family member, so its connection to energy has been well established for the past $\sim$150 years.
What all this means to the science of homeopathy is that Equation 6c is the operational one that must be followed (as best one can) and that the relevant thermodynamics is that which includes both levels of physical reality, including the deltron coupling medium. The still higher dimensional aspects of a truly operational RF\(^{11,12,2,6}\) can be set aside for the moment. For homeopathy, the simplest system needed is:

[D-space substance // deltrons // R-space substance ]

(Equation 7)

The procedure of dilution/succussion with archetypal intention, via Hahnemann and subsequent homeopathic practitioners, is particularly interesting when \(\mu_j\) falls into the negative region of Figure 1b. Then, three important considerations come into play: (1) assuming that D-space/R-space equilibrium for the solution existed prior to the D-space dilution process, the R-space conjugate pattern for j is still at full strength and exerts a real force, approximately proportional to \(\alpha_{\text{eff}}\), on the D-space domain. This force would be such as to stimulate some beneficial changes in the hydrogen bond network. (2) The D-space thermodynamic forces also try to resist the loss of entropy of mixing for j as the D-space dilution process continues. (3) The succussion process itself is a very chaotic process acting very non-linearly on this H-bond network so that more thermodynamically desirable H-bond network changes can be nucleated (provides the critical magnitude energy fluctuations needed for rearranging this H-bond network).

As stated elsewhere,\(^2\) this specific development of patches of unique order in the H-bond network can be thought of as involving the enhancement of deltron activation at each step of the succussion/dilution process so that \(\alpha_{\text{eff}}\) increases. The consequence of this increased force transfer from the R-space counterpart of j to the very dilute D-space water might be thought to be the creation of empty H-bond cages of close to the exact D-space biomolecule cage. The presence of this structure would tend to stabilize the R-space counterpart of j and provide the D-space configurational entropy of mixing, as a surrogate for the disappearing D-space biomolecule, j, to lower the free energy of the system and stabilize the entire process. The gas trapped in the nanobubbles might, in fact, enter these surrogate j-voids and mechanically stabilize such constructs.\(^2\)

Equation 3 is an experimental result from the author’s pH-work.\(^3-6\) From the preceding theoretic discussion concerning this duplex RF and its applications to homeopathy, one can generalize the analogue to Equation 3 for any experimentally measurable quality, \(Q_M\), in a partially, “psychoenergetically conditioned” space \((\alpha_{\text{eff}} \neq 0)\). This generalized result is:

\[
Q_M = Q_D(t) + \alpha_{\text{eff}}(t) Q_R(t)
\]

(Equation 8)

To illustrate how this works, consider the standard j-species homeopathic remedy preparation. Although dilution plus succussion (under archetypal intention) is involved, the author is presently unable to mathematically incorporate this succussion process into the analysis, but understand that it will be present. In addition, the author is presently unable to exactly calculate \(Q_R\) from \(Q_D(t)\) because so little is known about the true deltron activation function and its mathematical form. However, the author can still shine some light on the process by mathematically evaluating the equilibrium \(Q_R(t)\) in terms of \(Q_D(t)\). This procedure has been carried out in Appendix II, so see what it explains. If one takes the overly simplified case of continuous dilution, one finds that:

\[
c_j(t) = c_j(0) \exp(-\beta t)
\]

(Equation 9)

where \(c_j(0)\) is the \(t = 0\) j-species concentration in the water and \(\beta\) is the decay constant given by the product of the amount of dilution per cycle and the time, \(\tau\), per dilution/succussion cycle. Appendix II shows how to approximately calculate the equilibrium value of \(Q_R(t)\) for this \(Q_D(t)\). There, the final integral for \(Q_R(t)\) was not evaluated; however, one can easily see that it will lead to the product of a decaying function of \(\beta\) and t and a sinusoidally oscillating function of \(k, \beta, \text{ and } t\). This type of result is fully consistent with the
typical oscillatory experimental observations for homeopathy. In a later paper, the more exact step-function lowering of \( c_j \) will be treated (neglecting the succussion effect) and plots given of \( Q_D(t) \), \( Q_{E}(t) \), and \( Q_{M}(t) \) as a function of the relevant parameters.

REFERENCES


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APPENDIX I: A BRIEF REMINDER ABOUT SOME ASPECTS OF CHEMISTRY

For a simple chemical reaction between molecules A and B to produce C:

\[ A + B \rightarroweq C ; \quad K = \frac{a_c}{a_A a_B} \]  

(Equation I-1)

where \( K \) is the equilibrium constant for this reaction and \( a_j \) is the thermodynamic activity for the \( j \)-species. In turn, the standard free energy of a reaction is given by:

\[ \Delta G_0 = -RT \ln K = -RT [\ln a_c^\circ - (\ln a_A^\circ + \ln a_B^\circ)] \]  

(Equation I-2a)

The subscript zero refers to all components of the reaction being in their standard states at the given temperature, \( T \), and at one atmosphere pressure, whereas \( R \) is the gas constant (a per mole basis), the superscript \( e \) refers to the equilibrium state and \( \ln \) refers to logarithm to the base “\( e \).” Thus, the thermodynamic driving force for change is:

\[ \Delta G - \Delta G_0 = -RT [\ln a_c - (\ln a_A + \ln a_B)] \]  

(Equation I-2b)

Finally, the definition of \( a_j \) is:

\[ a_j = \gamma_j c_j \]  

(Equation I-3a)

so that:

\[ \ln a_j = \ln \gamma_j + \ln c_j \]  

(Equation I-3b)

Here, \( c_j \) = the concentration of \( j \)-species in the solvent; whereas \( \gamma_j \) = the thermodynamic activity coefficient of the \( j \)-species and, as such, \( \gamma_j \) relates to the sum of all the environmental effects stored in the solvent that act on this \( j \)-type of molecule.

To illustrate the importance of \( \gamma_j \), observe how it is altered as the intensive thermodynamic variables \( \mathbf{E} \) (electric field strength) and \( \mathbf{H} \) (magnetic field strength) are both increased from zero. The chemical potential, \( \mu_j \), with \( j \) as an electrically neutral species is:

\[ \mu_j = \mu_0j + kT \ln(\gamma_j c_j) - \frac{v}{2} \frac{d}{dc_j} \{ \varepsilon \mathbf{E}^2 + m \mathbf{H}^2 \} \]  

(Equation I-4a)

\[ = \mu_0j + kT \ln(\gamma_j c_j) \]  

(Equation I-4b)

where

\[ \dot{\gamma}_j = \gamma_j \exp \left[ \frac{-v}{2kT} \frac{d}{dc_j} \left( \varepsilon \mathbf{E}^2 + m \mathbf{H}^2 \right) \right] \]  

(Equation I-4c)

Here, \( \mu_0 \) is the standard state value of \( \mu \), \( v \) = molecule volume, \( \varepsilon \) = electrical permittivity, and \( m \) = magnetic permeability per cubic centimeter of solution. Thus, one sees that the \( \mathbf{E} \) and \( \mathbf{H} \) effects are convoluted into a modified activity coefficient effect. Once again, one sees this as an environmental effect.

As a second example, consider Equation 3 of the main text; that is:

\[ \text{pH}(t) = \text{pH}_1 + \zeta_{\text{eff}} \text{pH}_2(t) \]  

(Equation I-5a)

where \( \text{pH}_1 \) is always defined as:

\[ \text{pH}_1 = \log_{10} a_{H^+} = \log_{10} \gamma_{H^+} c_{H^+} \]  

(Equation I-5b)

One can always introduce the mathematical equality:

\[ \zeta_{\text{eff}} \text{pH}_2 = \log_{10} (10^{\zeta_{\text{eff}}} \text{pH}_2) \]  

(Equation I-5c)
Thus, inserting Equations I-5b and I-5c into Equation I-5a and combining, one has:

\[ \text{pH}(t) = \log_{10}(\gamma_{\text{H}^+} c_{\text{H}^+})(10^{\text{eq-pH}_2}) \]  

(Equation I-6a)

\[ \text{pH}(t) = \log_{10} \gamma_{\text{H}^+} c_{\text{H}^+} \]  

(Equation I-6b)

where

\[ \gamma_{\text{H}^+} = \gamma_{\text{H}^+} \cdot 10^{\text{eq-pH}_2} \]  

(Equation I-6c)

So, once again, from a conventional single space (level one) perspective, this anomalous pH(t) experimental behavior can be considered as a modified thermodynamic activity coefficient effect for H+ that does not go to unity for very dilute solutions.

**APPENDIX II: APPROXIMATION TO THE EVALUATION OF Q_{R^\text{Eq}}(t) from Q_{D}(t)**

In the dilution and complete mixing process for a j-species, D-space molecule with water (neglecting dissolved gases and succussion-induced structural change effects), one cycle takes time, \( \tau \), and involves dilution by a factor of \( 10^q \), where \( q \) is some number, 1, 2, \ldots and so on. Thus, assuming the process is *continuous* rather than *steplike*, the rate of change of j is given in the water by:

\[ \frac{dc_j}{dt} = -c_j/10^q \tau = -\beta c_j \]  

(Equation II-1a)

where:

\[ \beta = (10^q \tau)^{-1} \]  

(Equation II-1b)

This first-order differential equation, with \( c_j = c_j(0) \) at time \( t = 0 \) has the solution:

\[ c_j = c_j(0) \exp(-\beta t) \]  

(Equation II-1c)

Thus, \( c_j \) decays exponentially with time with decay constant \( \beta \) where, for \( q = 2 \) and \( \tau = 1 \) hour, \( \beta = 0.01 \) hours so the D-space concentration would drop, for this continuous process, by a factor of 2.7 in 100 hours, which illustrates the weakness of using this mathematical approximation. However, it makes the next step mathematically much simpler. Thus:

\[ Q_D(t) = c_j(0) e^{-\beta t} \]  

(Equation II-1d)

for the calculation of \( Q_{R^\text{Eq}}(t) \).

Neglecting the deltron activation function within the body of the integrals,\(^2\) and inserting it later via the use of \( \alpha_{\text{eff}} \), one has as an approximation:

\[ F_{R}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} Q_D(t') e^{ikt'} dt' \]  

(Equation II-2a)

and

\[ Q_D(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} F_R(k) e^{-ikt} dk \]  

(Equation II-2b)

where \( k \) is the reciprocal space coordinate corresponding with the D-space coordinate, \( t \), \( Q_D(t) \) is assumed to be zero from \( t' = -\infty \) to 0, and \( i = \sqrt{-1} \), the imaginary number symbol. With this procedure, one obtains \( F_R(k) = R_R \exp(i\theta_R) \), where \( R_R \) and \( \theta_R \) are mathematically complex amplitude and phase angle in R-space. What can actually be measurable is the intensity, \( I_R \), which is the mathematically real part of \( R_R \) and this is obtained via:

\[ I_R(k) = [F_R(k) F_R(k)]^{1/2} \]  

(Equation II-3)
where $F_R^a(k)$ is the complex conjugate of $F_R(k)$, obtained by replacing $-i$ for $+i$ and vice versa in $F_R(k)$. Finally:

$$Q_R^\text{Eq} = \int_{-\infty}^{+\infty} I_R^2(k) \, dk$$  \hspace{1cm} (Equation II-4)

Generally one neglects the negative domain of $k$ as not being physically real.

Inserting Equation (II-1d) into (II-2a) and evaluating leads to:

$$F_R(k) = \frac{c_j(0)[e^{(ik-\beta)t} - 1]}{\sqrt{2\pi} (ik - \beta)}; \quad F_R^*(k) = \frac{c_j(0)[e^{-(ik+\beta)t}]}{\sqrt{2\pi} (ik + \beta)}$$  \hspace{1cm} (Equation II-5)

so

$$I_R^2(k) = \frac{c_j(0)}{\sqrt{2\pi}} \left\{ \frac{e^{(ik-\beta)t} - 1}{(ik - \beta)(-ik - \beta)} \right\}$$  \hspace{1cm} (Equation II-6a)

$$= \frac{c_j(0)}{\sqrt{2\pi}} \left\{ e^{-2\beta t} + 1 - e^{-\beta t} (e^{ikt} + e^{-ikt}) \right\}$$  \hspace{1cm} (Equation II-6b)

$$= \frac{c_j(0)}{\sqrt{2\pi}} e^{-\beta t} \left\{ [e^{-\beta t} + e^{\beta t}] - [e^{-ikt} + e^{ikt}] \right\}$$  \hspace{1cm} (Equation II-6c)

$$= \frac{c_j(0)}{\sqrt{2\pi}} \left\{ \frac{\cosh (\beta t) - \cos (kt)}{(\beta^2 + k^2)} \right\}$$  \hspace{1cm} (Equation II-6d)

Now, for the last step, insert this value for $I_R^2(k)$ into Equation II-4 and one finds that:

$$Q_R^\text{Eq} = A(t) \int_0^\infty \left[ 1 - \frac{\cos (kt)}{\cosh (\beta t)} \right] \frac{dk}{(\beta^2 + k^2)}$$  \hspace{1cm} (Equation II-7a)

where:

$$A(t) = \frac{c_j(0)}{\sqrt{\pi/2}} e^{-\beta t} \cosh (\beta t)$$  \hspace{1cm} (Equation (I-7b))

Equations II-7 can be readily solved numerically.